Trigonometrikus egyenletek

1. Oldd meg a valós számok halmazán a következő egyenleteket!

a) $\sin x = \dfrac{\sqrt{2}}{2}$

$$\begin{aligned} x_1 &= \frac{\pi}{4} + k \cdot 2\pi \\ x_2 &= \frac{3\pi}{4} + k \cdot 2\pi \end{aligned}$$

b) $\sin x = \frac{1}{2}$

$$\begin{aligned} x_1 &= \frac{\pi}{6} + k \cdot 2\pi \\ x_2 &= \frac{5\pi}{6} + k \cdot 2\pi \end{aligned}$$

c) $\cos x = -1$

$$x = (2k+1) \cdot \pi$$

d) $\cos x = -0,5$

$$x = \pm\frac{2\pi}{3} + k \cdot 2\pi$$

e) $\sin x = -\frac{\sqrt{3}}{2}$

$$\begin{aligned} x_1 &= -\frac{\pi}{3} + k \cdot 2\pi \\ x_2 &= \frac{4\pi}{3} + k \cdot 2\pi \end{aligned}$$

2. Oldd meg a valós számok halmazán a következő egyenleteket!

a) $\sin 3x = 0$

$$x = k \cdot \frac{\pi}{3}$$

b) $\sin 2x = \frac{1}{2}$

$$\begin{aligned} x_1 &= \frac{\pi}{12} + k \cdot \pi \\ x_2 &= \frac{5\pi}{12} + k \cdot \pi \end{aligned}$$

c) $\cos 2x = \dfrac{1}{2}$

$$\begin{aligned} x_1 &= \frac{\pi}{6} + k \cdot \pi \\ x_2 &= -\frac{\pi}{6} + k \cdot \pi \end{aligned}$$

d) $\sin 2x = \dfrac{\sqrt{3}}{2}$

$$\begin{aligned} x_1 &= \frac{\pi}{6} + k \cdot \pi \\ x_2 &= \frac{\pi}{3} + k \cdot \pi \end{aligned}$$

e) $\cos\left(3x - \dfrac{\pi}{3}\right) = -\dfrac{1}{2}$

$$\begin{aligned} x_1 &= \frac{\pi}{3} + k \cdot \frac{2\pi}{3} \\ x_2 &= -\frac{\pi}{9} + k \cdot \frac{2\pi}{3} \end{aligned}$$

f) $\sin\left(4x - \dfrac{2\pi}{3}\right) = 1$

$$x = \frac{7\pi}{24} + k \cdot \frac{\pi}{2}$$

g) $\sin\left(5x - \dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

$$\begin{aligned} x_1 &= k \cdot \frac{2\pi}{5} \\ x_2 &= \frac{3\pi}{10} + k \cdot \frac{2\pi}{5} \end{aligned}$$

h) $\operatorname{tg}\left(x - \dfrac{\pi}{9}\right) = \sqrt{3}$

$$x = \frac{\pi}{2} + k \cdot \pi$$

i) $\sqrt{2}\cos 3x = 1$

$$\begin{aligned} x_1 &= \frac{\pi}{12} + k \cdot \frac{2\pi}{3} \\ x_2 &= -\frac{\pi}{12} + k \cdot \frac{2\pi}{3} \end{aligned}$$

j) $\sin \pi x = \dfrac{\sqrt{2}}{2}$

$$\begin{aligned} x_1 &= \frac{1}{4} + k \cdot 2 \\ x_2 &= \frac{3}{4} + k \cdot 2 \end{aligned}$$

3. Oldd meg az alábbi egyenleteket!

a) $\sin 8x = \sin 11x$

$$\begin{aligned} x_1 &= k \cdot \frac{2\pi}{3}, \\ x_2 &= \frac{\pi}{19} + k \cdot \frac{2\pi}{19} \end{aligned}$$

b) $\sin 6x = \sin 13x$

$$\begin{aligned} x_1 &= k \cdot \frac{2\pi}{7}, \\ x_2 &= \frac{\pi}{19} + k \cdot \frac{2\pi}{19} \end{aligned}$$

c) $\sin 4x = \sin 9x$

$$\begin{aligned} x_1 &= k \cdot \frac{2\pi}{5}, \\ x_2 &= \frac{\pi}{13} + k \cdot \frac{2\pi}{13} \end{aligned}$$

d) $\cos 9x = \cos 15x$

$$\begin{aligned} x_1 &= k \cdot \frac{\pi}{3}, \\ x_2 &= k \cdot \frac{\pi}{12} \end{aligned}$$

e) $\sin 10x = \sin 15x$

$$\begin{aligned} x_1 &= k \cdot \frac{2\pi}{5}, \\ x_2 &= \frac{\pi}{25} + k \cdot \frac{2\pi}{25} \end{aligned}$$

f) $\cos 5x = \cos 8x$

$$\begin{aligned} x_1 &= k \cdot \frac{2\pi}{3}, \\ x_2 &= k \cdot \frac{2\pi}{13} \end{aligned}$$

g) $\cos 8x = \cos 14x$

$$\begin{array}{l} x_1 = k \cdot \frac{\pi}{3}, \\ x_2 = k \cdot \frac{\pi}{11} \end{array}$$

h) $\cos 11x = \cos 14x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{3}, \\ x_2 = k \cdot \frac{2\pi}{25} \end{array}$$

i) $\sin 9x = \sin 13x$

$$\begin{array}{l} x_1 = k \cdot \frac{\pi}{2}, \\ x_2 = \frac{\pi}{22} + k \cdot \frac{\pi}{11} \end{array}$$

j) $\sin 5x = \sin 12x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{7}, \\ x_2 = \frac{\pi}{17} + k \cdot \frac{2\pi}{17} \end{array}$$

k) $\cos 6x = \cos 15x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{9}, \\ x_2 = k \cdot \frac{2\pi}{21} \end{array}$$

l) $\sin 10x = \sin 17x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{7}, \\ x_2 = \frac{\pi}{27} + k \cdot \frac{2\pi}{27} \end{array}$$

m) $\cos 10x = \cos 15x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{5}, \\ x_2 = k \cdot \frac{2\pi}{25} \end{array}$$

n) $\cos 11x = \cos 19x$

$$\begin{array}{l} x_1 = k \cdot \frac{\pi}{4}, \\ x_2 = k \cdot \frac{\pi}{15} \end{array}$$

o) $\cos 2x = \cos 9x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{7}, \\ x_2 = k \cdot \frac{2\pi}{11} \end{array}$$

p) $\sin 5x = \sin 10x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{5}, \\ x_2 = \frac{\pi}{15} + k \cdot \frac{2\pi}{15} \end{array}$$

q) $\sin 7x = \sin 15x$

$$\begin{array}{l} x_1 = k \cdot \frac{\pi}{4}, \\ x_2 = \frac{\pi}{22} + k \cdot \frac{\pi}{11} \end{array}$$

r) $\sin 5x = \sin 14x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{9}, \\ x_2 = \frac{\pi}{19} + k \cdot \frac{2\pi}{19} \end{array}$$

s) $\cos 6x = \cos 11x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{5}, \\ x_2 = k \cdot \frac{2\pi}{17} \end{array}$$

t) $\cos 11x = \cos 16x$

$$\begin{array}{l} x_1 = k \cdot \frac{2\pi}{5}, \\ x_2 = k \cdot \frac{2\pi}{27} \end{array}$$

4. Oldd meg az alábbi egyenleteket!

a) $\sin\left(2x + \frac{\pi}{3}\right) = \sin(5x)$

$$\begin{array}{l} x_1 = \frac{\pi}{9} + k \cdot \frac{2\pi}{3} \\ x_2 = \frac{2\pi}{21} + k \cdot \frac{2\pi}{7} \end{array}$$

b) $\sin\left(5x + \frac{\pi}{2}\right) = \sin(6x)$

$$\begin{array}{l} x_1 = \frac{\pi}{2} + k \cdot \frac{2\pi}{1} \\ x_2 = \frac{\pi}{22} + k \cdot \frac{2\pi}{11} \end{array}$$

c) $\cos\left(8x + \frac{\pi}{5}\right) = \cos(11x)$

$$\begin{array}{l} x_1 = \frac{\pi}{15} + k \cdot \frac{2\pi}{3} \\ x_2 = -\frac{\pi}{95} + k \cdot \frac{2\pi}{19} \end{array}$$

d) $\sin\left(4x + \frac{\pi}{7}\right) = \sin(5x)$

$$\begin{array}{l} x_1 = \frac{\pi}{7} + k \cdot \frac{2\pi}{1} \\ x_2 = \frac{2\pi}{21} + k \cdot \frac{2\pi}{9} \end{array}$$

e) $\cos\left(6x + \frac{\pi}{8}\right) = \cos(8x)$

$$\begin{array}{l} x_1 = \frac{\pi}{16} + k \cdot \frac{\pi}{1} \\ x_2 = -\frac{\pi}{112} + k \cdot \frac{\pi}{7} \end{array}$$

f) $\cos\left(2x + \frac{\pi}{5}\right) = \cos(6x)$

$$\begin{array}{l} x_1 = \frac{\pi}{20} + k \cdot \frac{\pi}{2} \\ x_2 = -\frac{\pi}{40} + k \cdot \frac{\pi}{4} \end{array}$$

g) $\sin\left(4x + \frac{\pi}{4}\right) = \sin(6x)$

$$\begin{array}{l} x_1 = \frac{\pi}{8} + k \cdot \frac{\pi}{1} \\ x_2 = \frac{3\pi}{40} + k \cdot \frac{\pi}{5} \end{array}$$

h) $\sin\left(5x + \frac{\pi}{3}\right) = \sin(6x)$

$$\begin{array}{l} x_1 = \frac{\pi}{3} + k \cdot \frac{2\pi}{1} \\ x_2 = \frac{2\pi}{33} + k \cdot \frac{2\pi}{11} \end{array}$$

i) $\sin\left(3x + \frac{\pi}{2}\right) = \sin(7x)$

$$\begin{array}{l} x_1 = \frac{\pi}{8} + k \cdot \frac{\pi}{2} \\ x_2 = \frac{\pi}{20} + k \cdot \frac{\pi}{5} \end{array}$$

j) $\sin\left(3x + \frac{\pi}{4}\right) = \sin(4x)$

$$\begin{array}{l} x_1 = \frac{\pi}{4} + k \cdot \frac{2\pi}{1} \\ x_2 = \frac{3\pi}{28} + k \cdot \frac{2\pi}{7} \end{array}$$

k) $\sin\left(6x + \frac{\pi}{2}\right) = \sin(9x)$

$$\begin{array}{l} x_1 = \frac{\pi}{6} + k \cdot \frac{2\pi}{3} \\ x_2 = \frac{\pi}{30} + k \cdot \frac{2\pi}{15} \end{array}$$

l) $\sin\left(6x + \frac{\pi}{4}\right) = \sin(8x)$

$$\begin{array}{l} x_1 = \frac{\pi}{8} + k \cdot \frac{\pi}{1} \\ x_2 = \frac{3\pi}{56} + k \cdot \frac{\pi}{7} \end{array}$$

m) $\sin\left(5x + \frac{\pi}{4}\right) = \sin(6x)$

$$\begin{array}{l} x_1 = \frac{\pi}{4} + k \cdot \frac{2\pi}{1} \\ x_2 = \frac{3\pi}{44} + k \cdot \frac{2\pi}{11} \end{array}$$

n) $\sin\left(9x + \frac{\pi}{3}\right) = \sin(13x)$

$$\begin{array}{l} x_1 = \frac{\pi}{12} + k \cdot \frac{\pi}{2} \\ x_2 = \frac{\pi}{33} + k \cdot \frac{\pi}{11} \end{array}$$

o) $\cos\left(2x + \frac{\pi}{4}\right) = \cos(6x)$

$$\begin{array}{l} x_1 = \frac{\pi}{16} + k \cdot \frac{\pi}{2} \\ x_2 = -\frac{\pi}{32} + k \cdot \frac{\pi}{4} \end{array}$$

p) $\cos\left(2x + \frac{\pi}{3}\right) = \cos(6x)$

$$\begin{array}{l} x_1 = \frac{\pi}{12} + k \cdot \frac{\pi}{2} \\ x_2 = -\frac{\pi}{24} + k \cdot \frac{\pi}{4} \end{array}$$

5. Oldd meg az alábbi egyenleteket!

a) $\sin 4x = \cos 7x$

$$\begin{array}{r} x_1 = \frac{\pi}{22} + k \cdot \frac{2\pi}{11} \\ x_2 = \frac{\pi}{6} + k \cdot \frac{2\pi}{3} \end{array}$$

b) $\cos 5x = \sin 9x$

$$\begin{array}{r} x_1 = \frac{\pi}{28} + k \cdot \frac{\pi}{7} \\ x_2 = -\frac{\pi}{8} + k \cdot \frac{\pi}{2} \end{array}$$

c) $\cos 3x = \sin 5x$

$$\begin{array}{r} x_1 = \frac{\pi}{16} + k \cdot \frac{\pi}{4} \\ x_2 = -\frac{\pi}{4} + k \cdot \frac{\pi}{1} \end{array}$$

d) $\sin 3x = \cos 6x$

$$\begin{array}{r} x_1 = \frac{\pi}{18} + k \cdot \frac{2\pi}{9} \\ x_2 = \frac{\pi}{6} + k \cdot \frac{2\pi}{3} \end{array}$$

e) $\sin 6x = \cos 9x$

$$\begin{array}{r} x_1 = \frac{\pi}{30} + k \cdot \frac{2\pi}{15} \\ x_2 = \frac{\pi}{6} + k \cdot \frac{2\pi}{3} \end{array}$$

f) $\cos 4x = \sin 5x$

$$\begin{array}{r} x_1 = \frac{\pi}{18} + k \cdot \frac{2\pi}{9} \\ x_2 = -\frac{\pi}{2} + k \cdot \frac{2\pi}{1} \end{array}$$

g) $\cos 6x = \sin 11x$

$$\begin{array}{r} x_1 = \frac{\pi}{34} + k \cdot \frac{2\pi}{17} \\ x_2 = -\frac{\pi}{10} + k \cdot \frac{2\pi}{5} \end{array}$$

h) $\cos 5x = \sin 7x$

$$\begin{array}{r} x_1 = \frac{\pi}{24} + k \cdot \frac{\pi}{6} \\ x_2 = -\frac{\pi}{4} + k \cdot \frac{\pi}{1} \end{array}$$

i) $\sin 6x = \cos 8x$

$$\begin{array}{r} x_1 = \frac{\pi}{28} + k \cdot \frac{\pi}{7} \\ x_2 = \frac{\pi}{4} + k \cdot \frac{\pi}{1} \end{array}$$

j) $\sin 2x = \cos 7x$

$$\begin{array}{r} x_1 = \frac{\pi}{18} + k \cdot \frac{2\pi}{9} \\ x_2 = \frac{\pi}{10} + k \cdot \frac{2\pi}{5} \end{array}$$

k) $\cos 4x = \sin 6x$

$$\begin{array}{r} x_1 = \frac{\pi}{20} + k \cdot \frac{\pi}{5} \\ x_2 = -\frac{\pi}{4} + k \cdot \frac{\pi}{1} \end{array}$$

l) $\sin 4x = \cos 8x$

$$\begin{array}{r} x_1 = \frac{\pi}{24} + k \cdot \frac{\pi}{6} \\ x_2 = \frac{\pi}{8} + k \cdot \frac{\pi}{2} \end{array}$$

m) $\cos 4x = \sin 7x$

$$\begin{array}{r} x_1 = \frac{\pi}{22} + k \cdot \frac{2\pi}{11} \\ x_2 = -\frac{\pi}{6} + k \cdot \frac{2\pi}{3} \end{array}$$

n) $\sin 4x = \cos 5x$

$$\begin{array}{r} x_1 = \frac{\pi}{18} + k \cdot \frac{2\pi}{9} \\ x_2 = \frac{\pi}{2} + k \cdot \frac{2\pi}{1} \end{array}$$

o) $\sin 2x = \cos 7x$

$$\begin{array}{r} x_1 = \frac{\pi}{18} + k \cdot \frac{2\pi}{9} \\ x_2 = \frac{\pi}{10} + k \cdot \frac{2\pi}{5} \end{array}$$

p) $\cos 4x = \sin 6x$

$$\begin{array}{r} x_1 = \frac{\pi}{20} + k \cdot \frac{\pi}{5} \\ x_2 = -\frac{\pi}{4} + k \cdot \frac{\pi}{1} \end{array}$$

q) $\cos 6x = \sin 9x$

$$\begin{array}{r} x_1 = \frac{\pi}{30} + k \cdot \frac{2\pi}{15} \\ x_2 = -\frac{\pi}{6} + k \cdot \frac{2\pi}{3} \end{array}$$

r) $\sin 5x = \cos 7x$

$$\begin{array}{r} x_1 = \frac{\pi}{24} + k \cdot \frac{\pi}{6} \\ x_2 = \frac{\pi}{4} + k \cdot \frac{\pi}{1} \end{array}$$

6. Oldd meg az alábbi egyenleteket!

a) $\sin^2 x = \frac{3}{4}$

$$x = \pm\frac{\pi}{3} + k \cdot \pi$$

b) $\cos x^2 = 1$

$$x = \pm\sqrt{k \cdot 2\pi}$$

c) $\operatorname{tg} x = -1$

$$x = -\frac{\pi}{4} + k \cdot \pi$$

d) $\operatorname{tg} x = -\frac{\sqrt{3}}{3}$

$$x = -\frac{\pi}{6} + k \cdot \pi$$

e) $\operatorname{tg} 5x = \operatorname{tg} x$

$$\begin{array}{r} x = k \cdot \frac{\pi}{4} \\ x \neq \frac{\pi}{2} + k \cdot \pi \\ x \neq \frac{\pi}{10} + k \cdot \pi \end{array}$$

f) $\cos 5x = \cos\left(x - \frac{\pi}{4}\right)$

$$\begin{array}{r} x_1 = \frac{\pi}{24} + k \cdot \frac{\pi}{3} \\ x_2 = -\frac{\pi}{16} + k \cdot \frac{\pi}{2} \end{array}$$

g) $2 \cdot \cos\left(4x - \frac{\pi}{2}\right) = -\sqrt{3}$

$$\begin{array}{r} x_1 = \frac{\pi}{3} + k \cdot \frac{\pi}{2} \\ x_2 = -\frac{\pi}{12} + k \cdot \frac{\pi}{2} \end{array}$$

h) $-2\sin\left(2x - \frac{3\pi}{4}\right) = \sqrt{2}$

$$\begin{array}{r} x_1 = \frac{\pi}{4} + k \cdot \pi \\ x_2 = \pi + k \cdot \pi \end{array}$$

7. Oldd meg a valós számok halmazán a következő, másodfokúra visszavezethető egyenleteket!

a) $\sin^2 x - 2\sin x = 0$

$$x = k \cdot \pi$$

b) $2\sin^2 x - 7\sin x + 3 = 0$

$$x_1 = \frac{\pi}{6} + k \cdot 2\pi$$
$$x_2 = \frac{5\pi}{6} + k \cdot 2\pi$$

c) $8\sin^2 x - 7\cos^2 x = 8$

$$x = \frac{\pi}{2} + k \cdot \pi$$

d) $\cos^2 x - \sin^2 x = \frac{1}{2}$

$$x_1 = \frac{\pi}{6} + k \cdot 2\pi$$
$$x_2 = \frac{5\pi}{6} + k \cdot 2\pi$$

e) $\sin^2 x - \cos^2 x = \cos x$

$$x_1 = \pi + k \cdot 2\pi$$
$$x_2 = \pm\frac{\pi}{3} + k \cdot 2\pi$$

f) $2\sin^2 x + 3\cos x = 0$

$$x = \pm\frac{2\pi}{3} + k \cdot 2\pi$$

g) $\cos^2 x - \sin x = 1$

$$x_1 = k \cdot \pi$$
$$x_2 = \frac{3\pi}{2} + k \cdot 2\pi$$

h) $-4\cos^2 x - 2\sin x + 3 = 0$

$$x_1 = \frac{3\pi}{10} + k \cdot 2\pi,$$
$$x_2 = \frac{7\pi}{10} + k \cdot 2\pi,$$
$$x_3 = -\frac{\pi}{10} + k \cdot 2\pi,$$
$$x_4 = \frac{11\pi}{10} + k \cdot 2\pi$$

i) $\sin^2 x + \cos x = 1$

$$x_1 = \frac{\pi}{2} + k \cdot \pi$$
$$x_2 = 0 + k \cdot 2\pi$$

j) $2\sin^2 x + 5 \cdot \cos x - 4 = 0$

$$x_1 = \frac{\pi}{3} + k \cdot 2\pi$$
$$x_2 = -\frac{\pi}{3} + k \cdot 2\pi$$

k) $4\cos^2 x + 17\sin x = 8$

$$x_1 = 14^\circ 29' + k \cdot 360^\circ$$
$$x_2 = 165^\circ 31' + (2k+1) \cdot 360^\circ$$

l) $\mathrm{tg}^2 x = 2\,\mathrm{tg}\, x$

$$x_1 = k \cdot \pi$$
$$x_2 = 63{,}43^\circ + k \cdot 180^\circ$$